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The Mental Mathematics of Trainee Teachers in the UK: Patterns and Preferences

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Abstract: The mathematics curriculum in the UK has changed dramatically over the last ten years and mental calculation strategies have attained a dominant position in the primary curriculum. Drawing on data from a study of trainee teachers' competence in mental mathematics, this study explores the implications of the change in curriculum focus for trainees' capacity to work 'connectedly' within the context of mental mathematics. The results and analysis of the test of trainee teachers' competence and their procedural preferences provide very little evidence of connected thinking. The differences and evidence of success associated with different operations suggest that they may experience and perceive mental mathematics, not as a whole, but as an assortment of disparate procedures. The study highlights the need to address the way in which calculation is covered on courses of initial teacher training.

Key words: Mental mathematics; Trainee teachers; Connected understanding; UK mathematics education

Introduction: the British Mathematics Curriculum

The National Numeracy Strategy (NNS) (DfEE, 1998) was introduced into British schools in 1999 because of fears about apparent falling standards in numeracy among school-leavers (Callaghan, 1987). There were also concerns that British schools compared badly with those in other countries (DfEE, 1998, Basic Skills Agency, 1997). A Task Group was set up in 1997 to review research and theory and thereby to explore the possibility of raising mathematical achievement. It led to the establishment of the NNS (Brown, Askew, Baker, Denvir, & Millet, 1998) which was incorporated in 2007 into the new Primary National Strategy (PNS) (DfES, 2007a). The PNS includes an online version of the updated NNS, called 'The Primary Framework for Literacy and Mathematics' (DfES, 2007a).

A new emphasis on mental mathematics

Mental mathematics was emphasized in the original NNS and in its most recent form, arguably as a result of some highly influential research taking place in the Netherlands (Beishuizen, 1997). The emphasis placed on developing mental calculation was radical in its implications and effects. For example, the NNS (DfEE, 1998) stated that children should not be taught a standard method of written calculation until they were able to 'add or subtract reliably any pair of two digit numbers in their heads' (DfEE, 1998, p7). However, although young children were expected to use oral methods, the use of pencil and paper was not prohibited. Informal jottings were to be encouraged and in the early years children were to be taught to record answers to problems. As children got older and began to use larger numbers, informal jottings could be used to assist their mental calculations.

Furthermore, children were now expected to deal with whole numbers. In comparison, until the implementation of the NNS (DfEE, 1998), children were taught to split numbers into tens and units and then add or subtract them separately. The stance adopted in the NNS (DfEE, 1998) was based on new evidence (Beishuizen, 1997) which suggested that children's understanding of the number system developed more effectively when they thought of numbers as a whole because they were more likely to make good use of estimation and approximation techniques.

The PNS (DfES, 2007a), the most recent manifestation of the NSS (DfEE, 1998), reasserts the importance of mental capability as is shown in the guidance paper written specifically about calculation (DfES, 2007b):

> As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. (p. 1)

The emphasis on mental capability was one of the biggest changes introduced in the NNS and a considerable amount of work has been undertaken to develop this aspect of children's mathematical understanding (Anghileri, 1999; Beishuizen, 1999; Treffers & Beishuizen, 1999; Thompson, 1997).

This work has led Beishuizen (1997) to point out some of the dangers inherent in the reliance on mental calculation. He argues that there is a difference between doing mental arithmetic in your head, and doing mental arithmetic with your head. He suggests that mental recall (i.e. memorizing number facts) is done in the head, whereas the mental strategies that lead to understanding are done with the head. Partly because results can be arrived at quickly, there is a danger, when talking about daily mental work (known and encouraged in the NSS (DfEE, 1998) as the "mental and oral starter") of focusing on doing work in the head, emphasizing the procedural at the expense of understanding.

The NNS (DfEE, 1998), and the PNS (DfES, 2007a) acknowledge the need to be cautious. The guidance paper for calculation suggests that the development of mental capability is more than drill and practice, procedural understanding and memorizing number facts; it is also about pupils making decisions and choices for themselves, using mental skills to monitor progress and to decide whether solutions make sense (DfES, 2007b).

The preparation of teachers

The NNS (DfEE, 1998) and the PNS (DfES, 2007a), having entrenched the development of pupils' mental capability within mathematics education at Key Stages 1 and 2[1] also took into account the implications for teachers and their preparation. The content of initial teacher education programmes was targeted through the introduction of Circular 4/98 (DfEE, 1998). This specified in considerable detail the knowledge, understanding and skills that all trainee teachers should have before the end of their course. In 2002, this circular was replaced with a document entitled 'Qualifying to teach' (Teacher Training Agency, 2003). The detailed listing of trainee teachers' required knowledge, understanding and skills was replaced by two sentences:

> They have a secure knowledge and understanding of the subjects they are trained to teach…. For Key Stage 1 and/or 2, they know and understand the curriculum for each of the National Curriculum core subjects, and the frameworks, methods and expectations set out in the National Numeracy Strategy …. (p. 8)

These publications suggest that there is an on-going requirement within teacher education to promote and assess the development of trainee teachers' own knowledge and understanding of mental mathematics as they prepare to meet the obligations and expectations of the NNS (DfEE, 1998) and the PNS (DfES, 2007a) .

In this context there is no known published research on the particular challenges involved in teaching mental mathematics. However, Beishuizen's (1997) concern about the distinction between working in the head and working with the head and the PNS's appeal for pupils to make decisions and choices for themselves, using mental skills to monitor progress and to decide whether solutions make sense (DfES, 2007b), are mirrored by concepts such as 'relational' understanding (Skemp, 1989) and 'connected knowledge' (Ma, 1999; Davis, 2001) which have emerged

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[1] In the UK, Key Stage 1 refers to pupil aged 5-7 and Key Stage 2 refers to pupils aged 8-11.

from general research on the knowledge requirements of teachers of mathematics conducted over the last two decades.

Skemp (1989) argued that understanding can be both relational and instrumental and if the aim is to develop relational understanding in children (that is, knowing both what to do and why), then teachers of mathematics must have relational understanding, too. Ball (1990) claimed that mathematics teachers need knowledge of the nature and discourse of mathematics enquiry and that their knowledge needs to be correct, connected and meaningful. Askew, Brown, Rhodes, Wiliam, and Johnson (1997) defined primary mathematics teachers' development in terms of their 'appreciation of the multifaceted nature of mathematical meaning' (p. 93).

Ma (1999) compared American and Chinese teachers and found that the Chinese education system encourages learning where problems are approached in a number of ways and where there is an expectation that any given result will be mathematically justified. She described this as 'profound' learning that is 'deep, broad and thorough' (p.121) and which produces connected knowledge. Connecting with more conceptually powerful ideas produces depth, connecting with concepts of similar power produces breadth and thoroughness is 'the capability to 'pass through' all parts of the field – to weave them together' (p. 121). Davis (2001) agrees. Arguing that connectedness is an integral part of mathematics classrooms, he claims that:

> Addition cannot be grasped without realising its relationship with subtraction and the way in which it operates within the set of natural numbers, the integers and ultimately the set of real numbers. (p. 136)

But he goes further when he suggests that:

> The connectedness of this discipline extends beyond the links between mathematical ideas as such. There are relationships to empirical concepts. For instance, we cannot exhaust the 'meaning' of subtraction merely by specifying the sets of numbers to which this operation may be applied, its relationship to addition, and so on. Something must also be said about the way in which it may be modelled in the 'real world'. It can be illustrated by means of the physical removal of objects from a group and by the physical comparison of one group of objects with another. (p. 137)

Connected thinking as described in the literature indicates an ease or familiarity with the relationships and connections between numbers, operations and objects. Problem solving relies on the construction of informal, bespoke strategies rather than the mechanical application of algorithms and standard methods, and multiple strategies are routinely used for testing and checking.

Logically, mental mathematics cannot be exempt from these ambitions. The implicit assumption is that if pupils are to experience mental mathematics in the way advocated in the literature and the PNS (DfES, 2007b), trainee teachers should develop the facility to work 'connectedly' (Ma, 1999) with regard to mental mathematics. This study is designed to test some of the elements within that assumption by seeking answers to the following specific questions:

1. In a cohort of trainee teachers what is the incidence of the use of informal strategies across a range of basic mental mathematics problems and when are they successful?

2. Do trainee teachers of mathematics rely on the use of algorithms or standard methods of written calculations to solve mental mathematics problems and to what effect?

3. To what extent do trainee teachers attempt to justify, mathematically, any given result, for example through the use of multiple checking methods?

The Study

The study's subjects were the 170 trainee teachers on a post graduate certificate of education teacher training programme lasting one year and focusing on the primary age group (Key Stages 1 and 2). Post Graduate Certificate of Education teacher training course. All participants had a first degree (or equivalent) in a subject directly relevant to the national curriculum in primary schools. In addition, to gain entry to the programme, trainees were required to have General Certificate of Secondary Education grade C or above (or equivalent) and to pass a basic mathematics test during the interview for a place on the course. The mathematics test included basic numeracy questions, together with problems requiring reasoning and proof. Test results showed that the cohort represented a broad range of ability and previous experience of mathematics.

The course began with two weeks' observation and task-focused experience in primary schools. Trainees then spent twelve weeks on a teaching programme within the University. This included modules on subject knowledge and pedagogic knowledge in the National Curriculum's core subjects of mathematics, English and science. Trainees then completed a six week teaching practice block in a primary school, followed by a further four weeks of University-based training. The final school placement of nine weeks was tapered to allow trainees to undertake an equivalent teaching timetable to that expected of a newly qualified teacher.

Two-thirds of the way through the course, prior to the final school placement, the whole cohort completed a test of mental mathematical competence (Table 1). The test consisted of twenty problems involving all four number operations and incorporating different numbers of digits, decimals and fractions. During the introduction to the test it was described as a mental test. However, trainees had a space in which to do jottings should they choose to do so. This was done to establish the preferred method of computation, to provide data on the methods and strategies used within and across items and to identify the frequency of use of multiple methods for individual items.

Trainees' use of formal or informal methods was of particular interest. Formal methods were defined as algorithms, that is, procedures for solving a problem in a finite number of steps which did not require an understanding of the process (e.g. solving division by fractions problems by inverting one fraction and multiplying, and using carrying, borrowing or decomposition for subtraction). Informal methods included rounding, adjusting and partitioning, where calculations took into account the numbers involved and decisions were made about the most appropriate calculation. For example, Item 5 (199 + 174) could be solved informally by rounding to 200, then adjusting (200 + 174 – 1).

Items were chosen so that their solution was relatively straightforward when using informal methods and rather more complicated when using standard methods of written calculation, i.e., formal algorithms, mentally. For example, Item 2 (1442 + 4739) was a relatively complicated calculation to perform mentally, using a formal algorithm, since it involved "carrying" several times as shown in Figure 1 and the potential for errors was large. It was relatively straightforward when using informal partitioning methods.

$$\begin{array}{ccccc} & 1 & 4 & 4 & 2 \\ + & 4 & 7 & 3 & 9 \\ & {\scriptstyle 1} & & {\scriptstyle 1} & \\ \hline & 6 & 1 & 8 & 1 \end{array}$$

Figure 1. Formal algorithm for Item 2

This was the justification for the study's assumption that trainees who solved the problems mentally (without using written working) used an informal method but it is acknowledged that this included cases where trainees relied on 'known facts' to which they had instant recall through memory.

Results

Items were marked and recorded according to whether the solution was correct, whether any written working was used and if so, whether it was a formal or informal method.

Table 1 shows a summary of the number of correct responses for each item and the type of method used. These data were analysed according to the study's specific research questions.

Incidence of the use of informal strategies

Table 2 shows the incidence of the attempted use of informal methods. The items are ordered, beginning with the item attempted most often, using informal methods.

Table 2 shows that the three questions prompting the greatest amount of informal working were those which involved multiplying or dividing by one or two digit numbers. In these cases, there was negligible recorded use of algorithms (Table 1). The fourth most commonly attempted item using informal methods was Item 18 (200 ÷ 25). Once again there was very little use of algorithms and none in relation to a related fact, 100 ÷ 25 = 4.

Item 20 (1.2 ÷ 0.2) was also frequently attempted using informal methods, but less than half of the trainees obtained the correct solution. The informal written strategies which trainees employed used knowledge of fractions, or inverse operations. Some trainees wrote the problem out again, using fractions $\frac{12}{10} \div \frac{2}{10}$ then rearranged to give $\frac{12}{10} \times \frac{10}{2} = 6$, or $\frac{6}{5} \div \frac{1}{5} = \frac{6}{5} \times \frac{5}{1} = \frac{30}{5} = 6$. When this approach was used, the correct solution was obtained. Another strategy used was to rearrange to give 0.2 × 5 = 6. Two of the three items which prompted the least amount of informal working involved multiplication and subtraction of fractions (Table 2).

Table 1
Responses and Incidence of Written and Informal Working by Item

Item No.	Item	% of trainees with correct solutions	Results link to attempted questions (not necessarily correct)		Results link to those solutions which were correct		
			% of trainees **attempting** each question using written working	% of informal methods out of those who **attempted** using written methods	% of correct solutions without using written working out of those who got it correct	% correct solutions using informal written methods out of those who got it correct	% correct solutions using some sort of informal method
1	95 + 46	95	40	43	57	16	73
2	1442 + 4739	89	87	20	9	16	25
3	$\frac{1}{2} + \frac{3}{8}$	82	72	9	19	5	24
4	0.4 + 2.8	94	35	10	62	3	65
5	199 + 174	92	58	41	39	23	62
6	49 – 18	92	38	40	57	14	70
7	2002 – 2485	63	50	19	35	6	41
8	$\frac{3}{4} - \frac{2}{3}$	64	78	3	6	0	6
9	2.58 – 1.29	77	77	14	14	8	22
10	7 × 8	92	6	100	86	5	92
11	32 × 20	93	50	78	47	36	83
12	155 + 156	93	52	38	46	18	64
13	52 × 34	63	94	45	0	22	22
14	$\frac{2}{3} \times \frac{5}{8}$	48	73	9	2	4	6
15	3.4 × 4.9	31	78	26	1	4	5
16	8 ÷ 2	98	1	0	97	0	97
17	42 ÷ 7	98	9	82	90	8	97
18	200 ÷ 25	88	15	78	75	9	84
19	$\frac{7}{8} \div \frac{1}{3}$	24	53	6	1	0	1
20	1.2 ÷ 0.2	49	30	7	32	13	45

Table 2
Responses and Incidence of the Attempted Use of Informal Methods

Item No.	Attempted, using informal methods
10	7×8
16	$8 \div 2$
17	$42 \div 7$
18	$200 \div 25$
20	$1.2 \div 0.2$
11	30×20
1	95 + 46
6	49 – 18
4	0.4 + 2.8
12	155 + 156
5	199 + 174
7	2002 – 2485
19	$\frac{7}{8} \div \frac{1}{3}$
13	52×34
15	3.4×4.9
3	$\frac{1}{2} + \frac{3}{8}$
9	2.58 – 1.29
14	$\frac{2}{3} \times \frac{5}{8}$
2	1442 + 4739
8	$\frac{3}{4} - \frac{2}{3}$

Successful use of informal methods

Table 3 shows these items in order of those solved correctly, using informal methods.

The same three items appear at the top of Tables 2 and 3, showing that the three most correct items were also solved informally. This relationship appeared to hold for all items suggesting a relationship between choice of strategy and success. The suggestion was further examined during analysis of the jottings provided on the test papers.

Table 3
Responses and Incidence of Correct Solution using Informal Methods

Item No.	Correct, using informal methods
16	8 ÷ 2
17	42 ÷ 7
10	7 × 8
18	200 ÷ 25
11	30 × 20
1	95 + 46
6	49 – 18
4	0.4 + 2.8
12	155 + 156
5	199 + 174
20	1.2 ÷ 0.2
7	2002 – 2485
2	1442 + 4739
3	$\frac{1}{2} + \frac{3}{8}$
13	52 × 34
9	2.58 – 1.29
14	$\frac{2}{3} \times \frac{5}{8}$
8	$\frac{3}{4} - \frac{2}{3}$
15	3.4 × 4.9
19	$\frac{7}{8} \div \frac{1}{3}$

Analysis and Discussion

Item type, choice of strategies and success

The relationship between the type of item, successful completion and reliance on written working, whether formal or informal, was complex. Item 13 (52 × 34) and Item 15 (3.4 × 4.9) (Table 1) illustrate this complexity. 63% of trainees correctly answered Item 13 and 94% of those who attempted it used some written working. Of those who chose to use written working, almost half used informal methods. The most popular informal method was one which involved some element of partitioning; this was done in a variety of ways, but the most frequent choice was (52 × 30) + (52 × 4). This item prompted the largest range of informal methods; below is an example of the types of strategies employed.

$$((34 \times 100) \div 2) + (34 \times 2)$$
$$((52 \times 10) \times 3) + (52 \times 4)$$
$$(10 \times 34 \times 5) + 34 + 34$$
$$((52 \times 20) + 520) + (52 \times 4)$$

5% of the cohort used the grid method for this item, but not always with success.

x	50	2
30	1500	60
4	200	8

In this case, the errors, which included entering incorrect numbers in the operant cells and entering 150 in the first cell rather than 1500, were not rectified through the application of checking procedures.

Of the 37% of trainees who were unable to solve this problem accurately, the most frequent error - the incorrect application of the distributive law – implied a lack of understanding of the relationship between the operation of multiplying in relation to the numbers:

$$52 \times 34 = (50 \times 30) + (2 \times 4)$$

The same errors were displayed for Item 15 (3.4×4.9). Only 31% of the trainees were able to solve this problem and very few did so without using written working. In total 78% tried to use written working, almost 75% of whom tried to use a formal algorithm such as that below:

	3	.	4	
×	4	.	9	
	1		3	
	3	.	0	6
1	3	.	6	0
1	6	.	6	6

Just under half of the trainees using this method gave 166.6 as an answer, implying a lack of connection between the operation of multiplying and place value and little recognition of the advisability of using alternative methods, for example estimation, to check.

The most commonly used informal strategy was to rewrite the problem without the decimal point, in effect multiplying both numbers by ten. However, many trainees divided the answer by ten rather than a hundred, again suggesting a lack of connection between the operation and place value. One trainee wrote that she didn't know how to complete this calculation accurately, but changed it to $3\frac{1}{2}$ multiplied by 5, to give an approximate solution.

As with the previous item, partitioning was a popular choice of strategy but once again the connection between the operation and the numbers was not fully appreciated and the distributive law was incorrectly applied.

$$3.4 \times 4.9 = (3 \times 4) + (0.4 \times 0.9)$$

The four items, Items 3, 8, 14 and 19, which prompted the greatest amount of formal written working or algorithms were those involving fractions.

For three out of four of the fraction problems, approximately three quarters of the trainees used a formal written method. The fraction problem which prompted the least amount of written working (53%) was a problem involving division of fractions (Item 19: $\frac{7}{8} \div \frac{1}{3}$). However, this was clearly the problem that trainees found the most difficult; only 24% were able to solve it although they had successfully solved fraction problems involving addition and subtraction. The most common error was to invert the wrong fraction, resulting in an inverted solution. Six trainees added comments to their test paper noting that they were unable to remember which fraction to invert, or that they could not remember the rule for dividing fractions. During overheard conversations after the test trainees confessed that they did not know whether or not 'to turn upside down and multiply', which fraction to invert and whether they needed to find a common denominator.

A common error for the problem that involved multiplying fractions was the practice of finding a common denominator, and then multiplying or dividing the numbers. For example, a typical solution for Item 14 ($\frac{2}{3} \times \frac{5}{8}$) is shown below:

$$\frac{2}{3} \times \frac{5}{8} = \frac{16}{24} \times \frac{15}{24}$$

Although at this stage, the calculation was not incorrect, few trainees starting with this strategy went on to complete the problem. Two trainees continued to attempt to solve the problem, but made mistakes in calculating 16 × 15 or 24 × 24.

Item 7 ($2002 - 2485$) was the only item involving a negative solution. 63% of trainees gave the correct solution and of those, over half relied on a formal written method. Some inverted the numbers, used an algorithm and then remembered to add a minus sign. 37% of trainees failed to solve this problem. Of the incorrect solutions, the majority wrote down 483, omitting the negative sign.

Across all items where written working was used, a formal algorithm was the most popular choice. In fact, it was used in 75% of the cases. The exceptions were those items involving knowledge of multiplication tables (Items 10, 11, 17 and 18). When a formal algorithm was not used, the most popular strategies were partitioning, the grid method, rounding and adjusting, use of knowledge of place value, and doubling. 32% of the cohort used partitioning to solve Item 13 (52 × 34) and a further 10% used the grid method. For Item 12 (155 + 156) 29% of the cohort used a formal algorithm (such as that shown in Figure 1) and 13% used doubling, either by doubling 150, then adding 11, or doubling 155 and adding 1.

Checking strategies / justifiable results

Using Ma's (1999) definition of connected thinking in mathematics, it was possible to interrogate the data further to determine if there was evidence to show that trainees knew when their solutions were mathematically justified.

Trainees were given as long as they needed to complete the test and were encouraged to check their work. However, there was very little evidence of solutions being checked for accuracy. Where some evidence was available, it was mainly because informal (written) strategies had been used initially, and formal algorithms had been squeezed into the remaining space, or indeed on the back of the paper.

In other cases, even when trainees were seemingly aware that their use of algorithms was defective, there was little evidence of attempts to use other strategies to check. For example, fewer than half the trainees gave a correct solution to Item 14 ($\frac{2}{3} \times \frac{5}{8}$). This was the question which prompted the most amount of crossed out working, implying that trainees had some level of understanding that either the answer or the process was incorrect. But there was no evidence of the substitution of other strategies.

The use of rote memory to recall number facts

The National Numeracy Strategy (DfEE, 1998) suggested that the ability to remember number facts and recall them without hesitation was a necessary element in the development of numeracy.

The results from some of the questions in this study shed some light on trainee teachers' use of rote memory to recall number facts. Some of the most commonly correctly-solved problems in the competence test were also the ones which prompted the least amount of written working (Table 1). These were Item 10 (7×8), Item 16 ($8 \div 2$) and Item 17 ($42 \div 7$). At least 90% of the trainees were able to solve these problems correctly without written working. For the purpose of this research, as was noted earlier, it was assumed that no written working implied the use of informal methods. However, in the case of these questions it is possible that trainees relied on the recall of facts memorized in primary school. All of the problems could have been solved using knowledge of multiplication tables without recourse to invented or informal methods and without making connections between multiplication and its inverse operation, division. So, since they were successful, would it matter if trainee teachers were relying on recalled facts rather than using informal methods? The evidence from Item 20 ($1.2 \div 0.2$) suggests that it did. Although the basic number fact required to solve the problem was extremely simple, only 49% of trainees were able to solve it correctly. This implies that for all but the most basic problems, rote knowledge of number facts is insufficient for their solution.

As has been reported above, the data provided examples of the use of what have been defined as 'formal' algorithms. There was also evidence that participants may have used some 'informal' methods as a set of rules, hence creating a new algorithm. The grid method was one such example. Its use in some circumstances, as was reported above, led to incorrect solutions because it was applied regardless of place value.

Characteristic strategies of successful trainees

The results for those trainees obtaining correct solutions for at least 85% of items show that their approaches to each item varied significantly, but the four items which prompted the most frequent use of formal algorithms were Item 2 ($1442 + 4739$), Item 9 ($2.58 - 1.29$), Item 13 (52×34), and Item 15 (3.4×4.9). This suggests that formal methods are likely to be used when the numbers are large, when more steps are involved during the use informal methods, or in the case of decimals.

Conclusions and Implications

The results and analysis of the test of trainee teachers' competence and their procedural preferences as revealed by formal and informal written methods provide very little evidence of connected thinking. For example, the use of multiplication and addition as if they are interchangeable involves a denial of the fundamental connection between these and other operations. Confusion about the inversion of fractions algorithm directly challenges the internal consistency of the multiplication and division operations. The significant differences and evidence of success associated with different operations suggest that the trainee teachers experience and perceive mathematics, not as a whole, but as an assortment of disparate procedures. This provides support for the proposal made by Frank (1990) and Foss and Kleinsasser (1996) that many teachers view mathematics as a collection of fragmented facts, procedures and right and wrong answers.

This conclusion highlights the need to address the way in which calculation is covered on courses of initial teacher training. On the basis of the results, it is difficult to be as confident as Murphy (2006) about the development of trainee teachers' connected mathematics thinking through the practice of teaching. The trainees in the study had had substantial school experience before taking the test and the results give at least some cause for concern, suggesting that a difference may exist between the rhetoric of pedagogical skills for relational teaching and what Murray (2006) refers to as the necessary underpinning reconstruction of trainee teachers' own subject understanding. The implication is that the calculation element of the course needs to make provision for the development of a 'fuller, deeper understanding of the number system and number operations and relations, and the way different interpretations of these interconnect' (Askew et al., 1997, p. 93).

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